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Electron and ion fluxes to a dust grain in atmospheric pressure plasma

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Abstract
Approximate expressions for the electron and ion fluxes to a dust grain under conditions $a \ll l \ll \lambda$ (a is the grain radius, l is the mean free path of electrons or ions and λ is the screening length) are presented. Such conditions can be realized, for example, in a plasma at atmospheric pressure and temperature $T \sim 0.1$ eV for grains of (sub)micron size. In two limit cases $l \ll a, \lambda$ and $l \gg a, \lambda$, the expressions for the fluxes are well known. In the intermediate case, the motion of an electron or ion in the spherical layer $a < r < a + l$ is considered as collisionless in the Coulomb potential, while for $r > a + l$ the drift-diffusion approximation is used. The obtained expression for the electron flux is applied for setting the boundary condition near the dust grain in the case of a thermionic plasma.

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1. Introduction

The flux of ions or electrons to (from) a dust grain is one of the most important quantities in dusty plasma theory. If the electron emission from the grain surface is negligible, the grain acquires negative charge ($Z_d < 0$) due to higher electron mobility. However, if the emission processes (thermionic, secondary or photoelectron emission) dominate, the grain charge may become positive ($Z_d > 0$). In the present paper, we consider the fluxes of particles attracted and repulsed to a charged dust grain (corresponding quantities will be marked by subscripts 'att' and 'rep', respectively). If $Z_d < (>)\, 0$ then the ions are attracted (repulsed) particles, while the electrons are repulsed (attracted) ones.

Usually experiments with dusty plasmas are performed in low-pressure discharges, where ion (electron) mean free path l exceeds the dust grain radius a and screening length λ. Under such conditions, one can use the known orbit motion limited (OML) approximation [1].

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In this case, the fluxes are expressed by equations

$$\Gamma_{\text{att}}^{\text{OML}} = \pi a^2 n_0 v_T \left(1 + \frac{|e\varphi_s|}{T}\right), \qquad \Gamma_{\text{rep}}^{\text{OML}} = \pi a^2 n_0 v_T \exp\left(-\frac{|e\varphi_s|}{T}\right). \tag{1}$$

Here, n_0 is the particle number density of the corresponding species at large distance ($r \gg \lambda$) where the electrostatic potential $\varphi = 0$; T is the temperature of the particles considered; $v_T = (8T/\pi m)^{1/2}$ is the mean thermal velocity; m is the particle mass; e is the elementary charge; $\varphi_s = \varphi(a)$ is the grain surface potential. In the opposite limiting case $l \ll a, \lambda$, the dust grain surface can be regarded as a flat one and the drift-diffusion approximation is valid down to the surface. Then

$$\Gamma_{\text{att}}^{\text{s}} = \pi a^2 n_{\text{s}} v_T, \qquad \Gamma_{\text{rep}}^{\text{s}} = \pi a^2 n_{\text{s}} v_T, \tag{2}$$

where $n_s = n(a)$ is the number density at the dust grain surface.

In the present paper, we consider the intermediate case

$$a \ll l \ll \lambda. \tag{3}$$

Such conditions can be realized, for example, in a plasma at atmospheric pressure and temperature $T \sim 0.1$ eV for dust grains of micron and submicron size.

2. Theoretical model

When the condition (3) is satisfied, we can regard the sheath $a < r < a + l$ around a dust grain as a collisionless region for charged (attracted or repulsed) particles, and the interaction potential can be approximated by the Coulomb, $\varphi = Z_d e/r$. For a particle incoming to the collisionless sheath with velocity $\mathbf{v}$, the minimal distance of the trajectory from the grain centre is [2]

$$r_{\min} = \frac{|Z_d|e^2}{2|E|}\left|1 \mp \sqrt{1 + \frac{2EM^2}{mZ_d^2e^4}}\right|, \qquad E = \frac{mv^2}{2} \mp \frac{|Z_d|e^2}{a+l}, \qquad M = m(a+l)v\sin\theta,$$

where the upper (lower) sign corresponds to the attracted (repulsed) particle and $\cos\theta = -\mathbf{r}\mathbf{v}/rv$ (θ is the angle between the velocity vector $\mathbf{v}$ and the direction to the grain centre). Only particles for which $r_{\min} \leqslant a$ reach the grain surface. This inequality holds if

$$\sin\theta \leqslant \frac{a}{a+l}\sqrt{1 \pm \frac{2|Z_d|e^2 l}{mv^2 a(a+l)}}. \tag{4}$$

For $r > a + l$, we use the drift-diffusion approximation. The flux density of particles (electrons or ions) incoming to the collisionless sheath (across the boundary $r = a + l$) with velocity between v and $v + \mathrm{d}v$ and direction inside the solid angle $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$ ($\theta < \pi/2$, ϕ is the azimuth angle) is $n_l v\cos\theta f(v)\,\mathrm{d}v\,\mathrm{d}\Omega$, where $f(v)$ is the velocity distribution function, n_l is the particle number density at $r = a + l$. After integration over $\mathrm{d}v\,\mathrm{d}\Omega$, taking into account (4), we obtain the flux to the grain

$$\Gamma = 4\pi(a+l)^2 n_l \int vq(v)f(v)\,\mathrm{d}v = 4\pi(a+l)^2 n_l\langle vq\rangle, \tag{5}$$

where $q(v)$ is the probability of reaching the grain for a particle of velocity v (averaged over directions):

$$q(v) = \frac{1}{4\pi}\int\cos\theta\,\mathrm{d}\Omega = \frac{1}{2}\int_0^{\theta_m}\cos\theta\sin\theta\,\mathrm{d}\theta = \frac{\sin^2\theta_m}{4},$$

$$\sin\theta_m = \min\left\{1, \frac{a}{a+l}\sqrt{1 \pm \frac{2|Z_d|e^2 l}{mv^2 a(a+l)}}\right\}.$$

For attracted and repulsed particles we have, respectively,

$$q_{\mathrm{att}}(v) = \begin{cases} 1/4, & v \leqslant v_1, \\ \dfrac{a^2}{4(a+l)^2}\left(1 + \dfrac{2|Z_d|e^2 l}{mv^2 a(a+l)}\right), & v > v_1, \end{cases} \quad v_1 = \sqrt{\frac{2|Z_d|e^2 a}{m(a+l)(2a+l)}},$$

$$q_{\mathrm{rep}}(v) = \begin{cases} 0, & v \leqslant v_{\mathrm{min}}, \\ \dfrac{a^2}{4(a+l)^2}\left(1 - \dfrac{2|Z_d|e^2 l}{mv^2 a(a+l)}\right), & v > v_{\mathrm{min}}, \end{cases} \quad v_{\mathrm{min}} = \sqrt{\frac{2|Z_d|e^2 l}{ma(a+l)}}.$$

All the attracted particles with low velocities $v \leqslant v_1$ (for $\theta < \pi/2$) fall to the grain ($\sin\theta_m = 1$), while a part of the fast particles, $v > v_1$, go away from the grain due to the increasing role of the centrifugal repulsion. But only the sufficiently fast repulsed particles, $v > v_{\mathrm{min}}$, can reach the grain. So, we find

$$\langle v q_{\mathrm{att}}\rangle = \int_0^\infty v q_{\mathrm{att}}(v) f(v)\,\mathrm{d}v, \qquad \langle v q_{\mathrm{rep}}\rangle = \int_{v_{\mathrm{min}}}^\infty v q_{\mathrm{rep}}(v) f(v)\,\mathrm{d}v, \tag{6}$$

and the corresponding fluxes are given by equation (5). We use the Maxwell distribution function for electrons and ions. For positive dust grains, it is a good approximation both for electrons and ions because the diffuse and drift flows are directed in opposite directions and substantially compensate each other. For negative grains, these arguments are valid only for electrons. However, the resonance charge exchange cross section of ions usually significantly exceeds the elastic cross section. In such cases, it is possible to use the Maxwell distribution with neutral gas temperature also for ions. Thus, for the Maxwell distribution, we obtain

$$\Gamma_{\mathrm{att}} = \pi(l+a)^2 n_l v_T \left[1 - \frac{l(l+2a)}{(l+a)^2}\exp\left(-\frac{|Z_d|e^2 a}{(l+a)(l+2a)T}\right)\right], \tag{7}$$

$$\Gamma_{\mathrm{rep}} = \pi a^2 n_l v_T \exp\left(-\frac{|Z_d|e^2 l}{a(l+a)T}\right). \tag{8}$$

Let us consider the relation between (7), (8) and (1), (2). We note that the first of inequalities (3) is necessary only for regarding the sheath $a < r < a+l$ as a collisionless one, and it is not used for the derivation of (7) and (8). Therefore, since the limit process $l/a \to 0$ is formally possible, as a result $n_l \to n_s$ and (7), (8) reduce to (2). In the opposite limit $a/l \to 0$, equation (8) reduces to the second one of (1), if we denote $Z_d e/a = \varphi_s$ and replace n_l by n_0. However for the reduction of equation (7) to the first of (1), we should also omit the terms higher than the linear in the exponent expansion.

3. Boundary condition near a dust grain

Under stationary conditions ($\mathrm{d}Z_d/\mathrm{d}t = 0$), the total current to a grain is equal to zero. Therefore $\Gamma_i - \Gamma_e + \Gamma_{e,\mathrm{em}} = 0$, where $\Gamma_{e,\mathrm{em}}$ is the electron flux from the grain surface due to the emission processes. For simplicity, we consider here the case of isothermal thermionic plasma assuming that the gas is not ionized. Then, we have

$$\Gamma_e - \Gamma_{e,\mathrm{em}} = 0. \tag{9}$$

In the drift-diffusion approximation ($r > a + l_e$) using the Einstein relation, we obtain

$$j_e(r) = \mu_e n_e\,\mathrm{d}\varphi/\mathrm{d}r - D_e\,\mathrm{d}n_e/\mathrm{d}r = 0, \qquad n_e = n_{e,0}\exp(e\varphi/T), \tag{10}$$

where $j_e(r)$ is the electron flux density, μ_e and D_e are the electron mobility and diffusivity, respectively. Then, the distribution of potential around a grain is defined by the Poisson–Boltzmann equation

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\varphi}{\mathrm{d}r}\right) = 4\pi e n_{e,0}\exp\left(\frac{e\varphi}{T}\right), \tag{11}$$

which is valid approximately for $r > a + l_e$. If the grain charge is sufficiently large, we can use the cell model [3] and solve (11) with the boundary conditions $\varphi(R_c) = 0$ and $\mathrm{d}\varphi/\mathrm{d}r|_{r=R_c} = 0$ at the cell radius $R_c = (4\pi n_d/3)^{-1/3}$, where n_d is the grain concentration. The solution of (11) can be found analytically [4] or numerically. The electron density is defined by (10) and the grain charge is $Z_d = -[(a + l_e)^2/e]\,\mathrm{d}\varphi/\mathrm{d}r|_{r=a+l_e}$, however, the value $n_{e,0} = n_e(R_c)$ is usually unknown; then we can use the boundary condition at $r = a + l_e$ obtained from (9). The flux of the thermionic emission is defined by the Richardson–Dushman formula [3]:

$$\Gamma_{e,\mathrm{em}} = \Gamma_{\mathrm{RD}} = 16\pi^2 m_e T^2 (2\pi\hbar)^{-3} a^2\,\mathrm{e}^{-W/T}, \tag{12}$$

where W is the work function. Equating (12) with (7), we get

$$n_{e,l} = \frac{2a^2(m_e T/2\pi\hbar^2)^{3/2}\,\mathrm{e}^{-W/T}}{(l_e + a)^2 - l_e(l_e + 2a)\exp\left(-\frac{Z_d e^2 a}{(l_e+a)(l_e+2a)T}\right)}. \tag{13}$$

Next, we apply this approach for evaluation of the charge of a CeO_2 grain in air for experimental conditions [5]: $a = 0.4$ μm, $n_d = 5 \times 10^7$ cm^{-3}, $W = 2.75$ eV, $T = 1700$ K, $\lambda \approx 20$ μm, $l_e \approx 3$ μm, $n_e = 2.5 \times 10^{10}$ cm^{-3}, $n_i \ll n_e$. Calculation with the boundary condition (13) gives $Z_d \approx 750$, that is in satisfactory agreement with the experimental value $Z_d = 500$ (setting the boundary condition at $r = a$ gives the noticeably lower value of about 230).

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